A 6-DOF Simulation of the Apollo 11 Command Module Entry

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This report provides a 6DOF simulation of the Apollo 11 Command Module entry and descent with a commanded bank angle profile from the original Apollo 11 flight data. We provide simulation results and a comparison with the Apollo 11 flight data.

Nomenclature

C_L	=	lift coefficient
C_L	=	drag coefficient
d	=	reference length
S	=	reference area
q_∞	=	free-stream dynamic pressure
RCS	=	reaction control system
$GRAM$	=	global reference atmospheric model
ECI	=	earth-centered inertial coordinate system (earth-centered celestial coordinate system)
ECEF	=	earth-centered earth-fixed coordinate system (terrestrial coordinate system)
ENU	=	east, north, up local coordinate system

I. Introduction

The Apollo 11 mission was the first American spaceflight to land on the moon. The Apollo command module (CM) Columbia was one of three parts of the Apollo spacecraft. The Earth reentry portion of the mission occurs with the astronauts in the CM. In this report, we provide a 6 degrees of freedom (DOF) simulation of the CM reentry from Earth entry interface. The entry phases of the Apollo CM are as follows. First the vehicle enters entry interface and follows a lift vector up capture until the altitude was decreasing at a rate less than $700 ft/s$. Then, the HUNTEST phase begins where the lift vector is commanded to point down until the predicted and desired range were within 25 nautical miles. Then the UPCONTROL phase begins where the bank angle is controlled to reach a skip velocity vector that ensures the predicted and calculated range are equal. Then, the final phase controls the bank angle to steer the CM to the landing target. The drogue parachutes deploy when the velocity falls below $1000 ft/s$ followed by the main parachutes. In this report we provide closed loop control of the bank angle profile from the Apollo 11 mission. A future

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improvement would be to declare the geodetic target location and solve an optimal control problem to select our own bank angle profile. This would ensure the landing zone is much closer to the target.

II. Approach

A. Equations of Motion

For our approach we write the equations of motion for a rigid body using the unit quaternion for representing orientation from J2000 ECI to the vehicle body frame. The states of the vehicle are inertial position $\boldsymbol{p}$, velocity $\boldsymbol{v}$ in the body frame, body angular rates $\boldsymbol{\omega}$, and quaternion for orientation $\boldsymbol{q}$. The equations of motion are as follows:

$$
\begin{aligned}
\dot{\boldsymbol{p}} &= \boldsymbol{R}_B^{ECI}\boldsymbol{v} \\
\dot{\boldsymbol{v}} &= \frac{1}{m}\boldsymbol{F} - \boldsymbol{\omega} \times \boldsymbol{v} \\
\dot{\boldsymbol{\omega}} &= \boldsymbol{J}^{-1}(\boldsymbol{M} - \boldsymbol{\omega} \times \boldsymbol{J}\,\boldsymbol{\omega}) \\
\dot{\boldsymbol{q}} &= \frac{1}{2}\boldsymbol{\Omega}(\boldsymbol{\omega})\boldsymbol{q}
\end{aligned}
\tag{1}
$$

where

$$
\Omega(\omega) = \begin{bmatrix} 0 & -\omega_x & -\omega_y & -\omega_z \\ \omega_x & 0 & -\omega_z & \omega_y \\ \omega_y & \omega_z & 0 & -\omega_x \\ \omega_z & -\omega_y & \omega_x & 0 \end{bmatrix} \tag{2}
$$

and $\boldsymbol{J}$ is the moment of inertia tensor. The external forces and moments from gravitational, aerodynamic, and thrusters are captured in $\boldsymbol{F}$ and $\boldsymbol{M}$.

B. Aerodynamics

The aerodynamic forces and moments are obtained from body frame force and moment coefficients. The coefficients are C_A for axial, C_N for normal, C_Y for side force, C_l for roll, C_m for pitching moment, and C_n for yawing moment.

The relationship between coefficients and their respective forces and moments are

$$
\begin{aligned}
C_A &= \frac{F_A}{q_\infty S} \\
C_N &= \frac{F_N}{q_\infty S} \\
C_Y &= \frac{F_Y}{q_\infty S} \\
C_l &= \frac{M_l}{q_\infty S d} \\
C_m &= \frac{M_m}{q_\infty S d} \\
C_n &= \frac{M_n}{q_\infty S d}.
\end{aligned}
\tag{3}
$$

The coefficients are extracted as a function of angle of attack and sideslip angle from [1] for a Mach number of $M = 9$. The data points are fitted to a polynomial function of degree 9 which is computed in the simulation. An improvement to this approach is to create a lookup table that also considers Mach number as an independent variable. The reference length $d = 3.91$ m is defined by the largest diameter of the capsule. The reference area $S = 12.0\,\mathrm{m}^2$ is defined by the area of the circle formed by the largest diameter.

C. bank angle control

The commanded bank angle profile is extracted from [2].

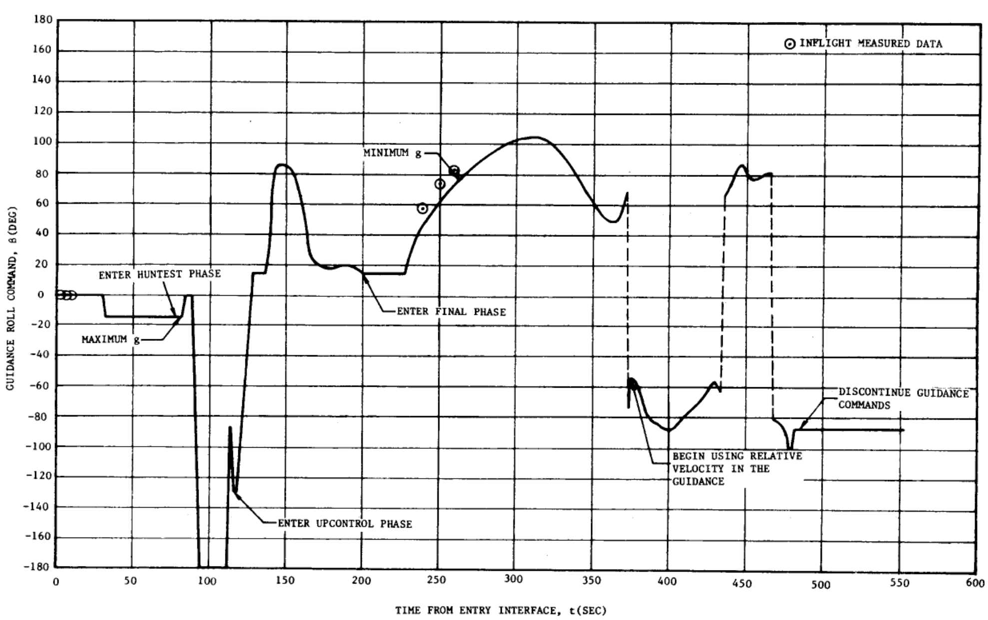

Fig. 1 Commanded bank angle profile [2].

To track the commanded bank angle we do proportional closed-loop feedback on the actual bank angle. The bank angle error is calculated from a wrapper function that output the angle between the actual and commanded bank angle between $[-\pi, \pi]$.

$$
\begin{aligned}
\boldsymbol{\sigma} &= [cos(\sigma), sin(\sigma), 0]^T \\
\boldsymbol{\sigma}_c &= [cos(\sigma_c), sin(\sigma_c), 0]^T \\
z_c &= [0, 0, 1] \\
\epsilon_c &= \text{atan2}((\boldsymbol{\sigma} \times \boldsymbol{\sigma}_c) \cdot z_c, \boldsymbol{\sigma} \cdot \boldsymbol{\sigma}_c)
\end{aligned}
\tag{4}
$$

We also implement body angular rate damping as in the original Apollo 11 CM. The vehicle moments from the RCS thrusters are therefore defined by

$$
\begin{aligned}
M_l &= 550(\epsilon_c) - 2500\omega_x \\
M_m &= -2000\omega_y \\
M_n &= -2000\omega_z.
\end{aligned}
\tag{5}
$$

The forces from the thrusters in the translational axis is neglected. Also there is no throttling of the RCS thrusters on the Apollo CM. However, in our approach we simplified the RCS model for the purpose of time. There is however four thrusters for each axis rotation. Two for negative and two for positive rotation. Therefore, there are 3 possible states for each axis - off, one on, or two on. A better approach is to implement a control strategy to control the RCS thrusters without throttling. This improves the fidelity of the model, but for the time constraint of this project this could not be implemented.

III. Results and Discussion

For this simulation we use the NASA Global Reference Atmospheric Model (GRAM) Suite for extracting density from altitude. For the gravity we use the GeographicLib library and the Earth Gravity Model 1996 (EGM96) dataset. We also use GeographicLib for transformations between geocentric and geodetic coordinates. For transformations between geocentric celestial (ECI) to terrestrial coordinate systems (ECEF) we use the Standards of Fundamental Astronomy library that uses the IAU 2006/2000A precession-nutation model. The following are the results of the simulation from the code attached to this report. In the next section we provide plots from the Apollo 11 flight data report to verify and discuss the comparison between the simulation.

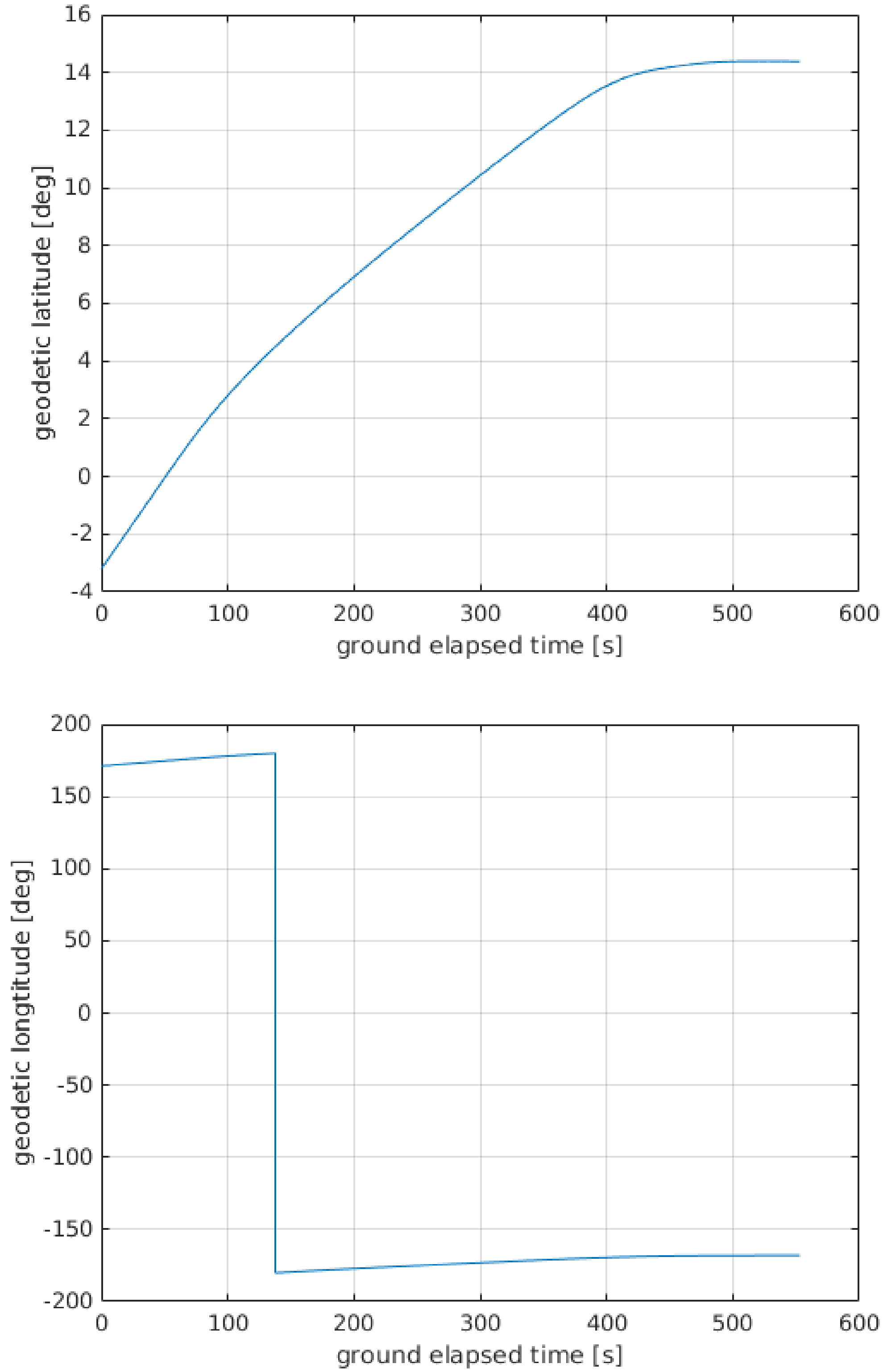

Fig. 2 Latitude and longitude plots from simulation.

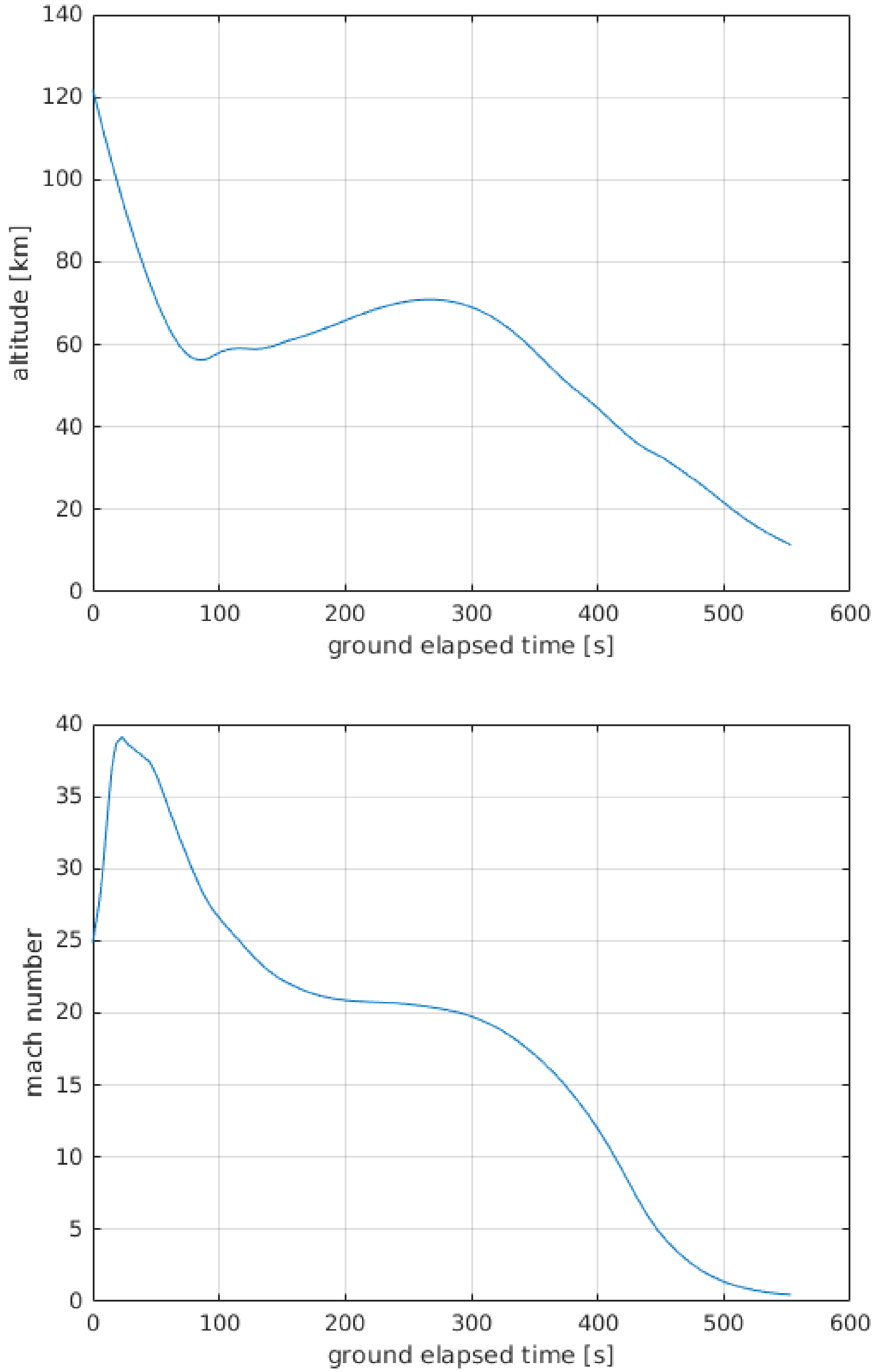

Fig. 3 Altitude and Mach number plot from simulation.

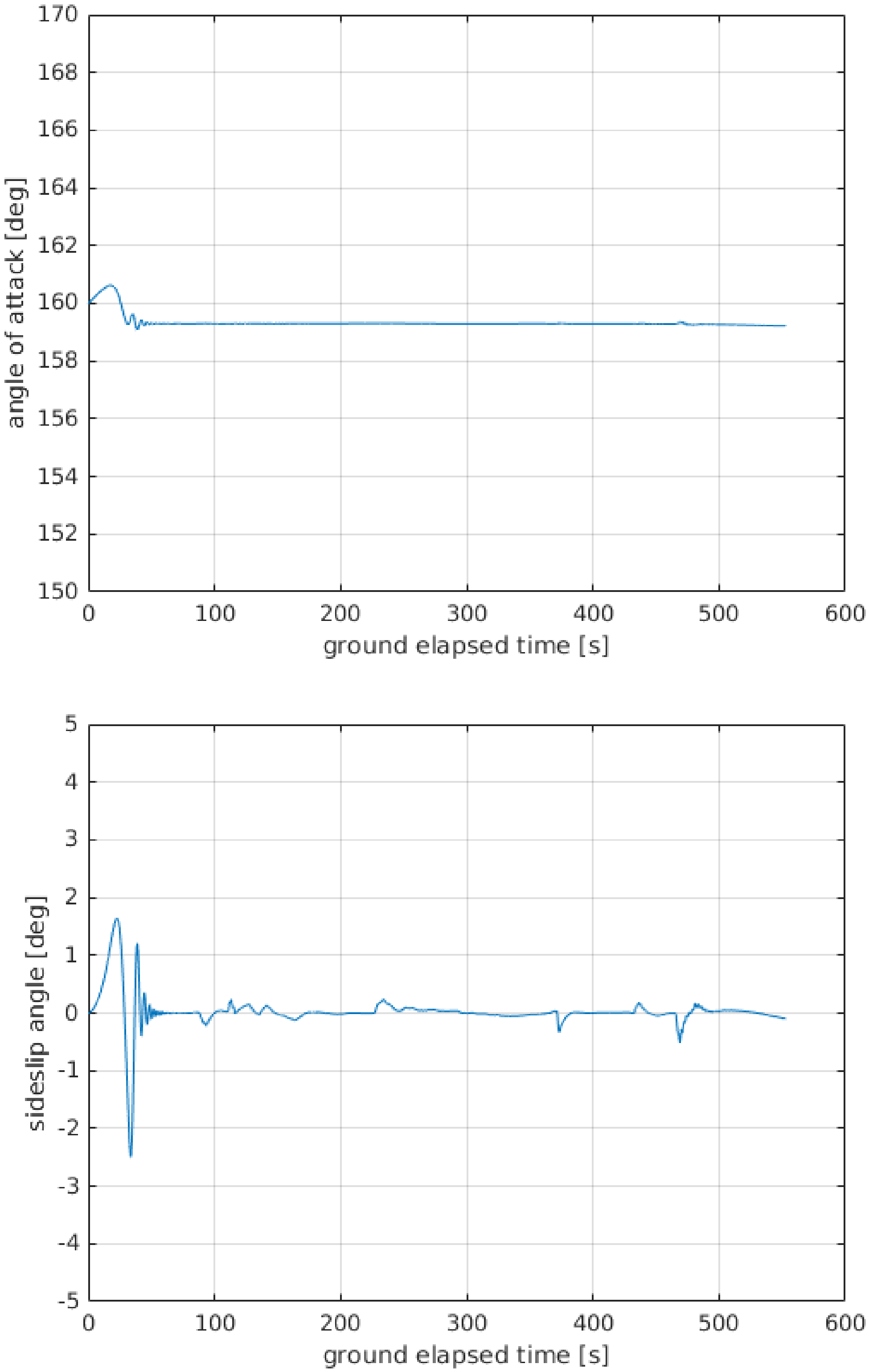

Fig. 4 Angle of attack and sideslip angle plot from simulation.

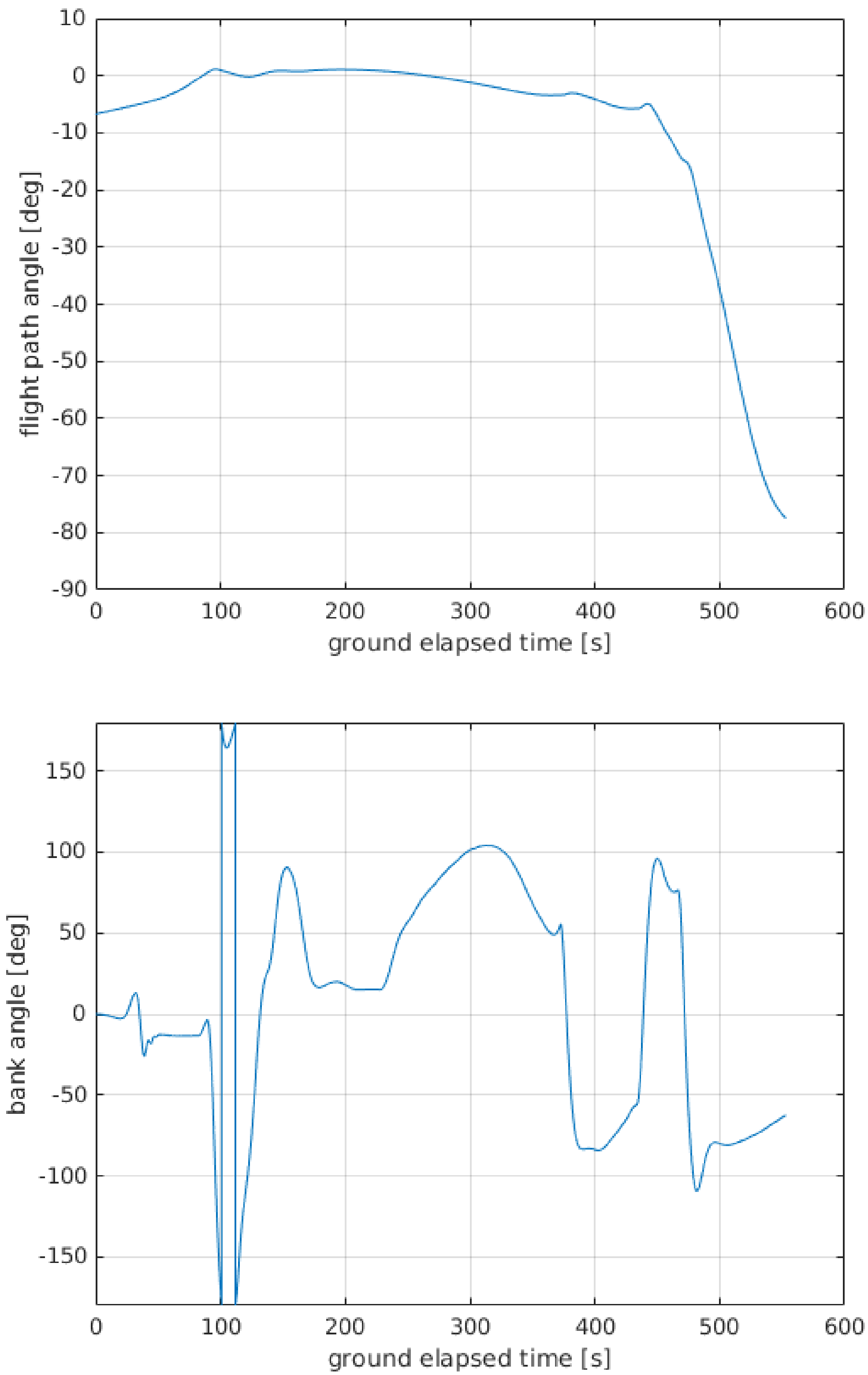

Fig. 5 Flight path angle and bank angle plot from simulation.

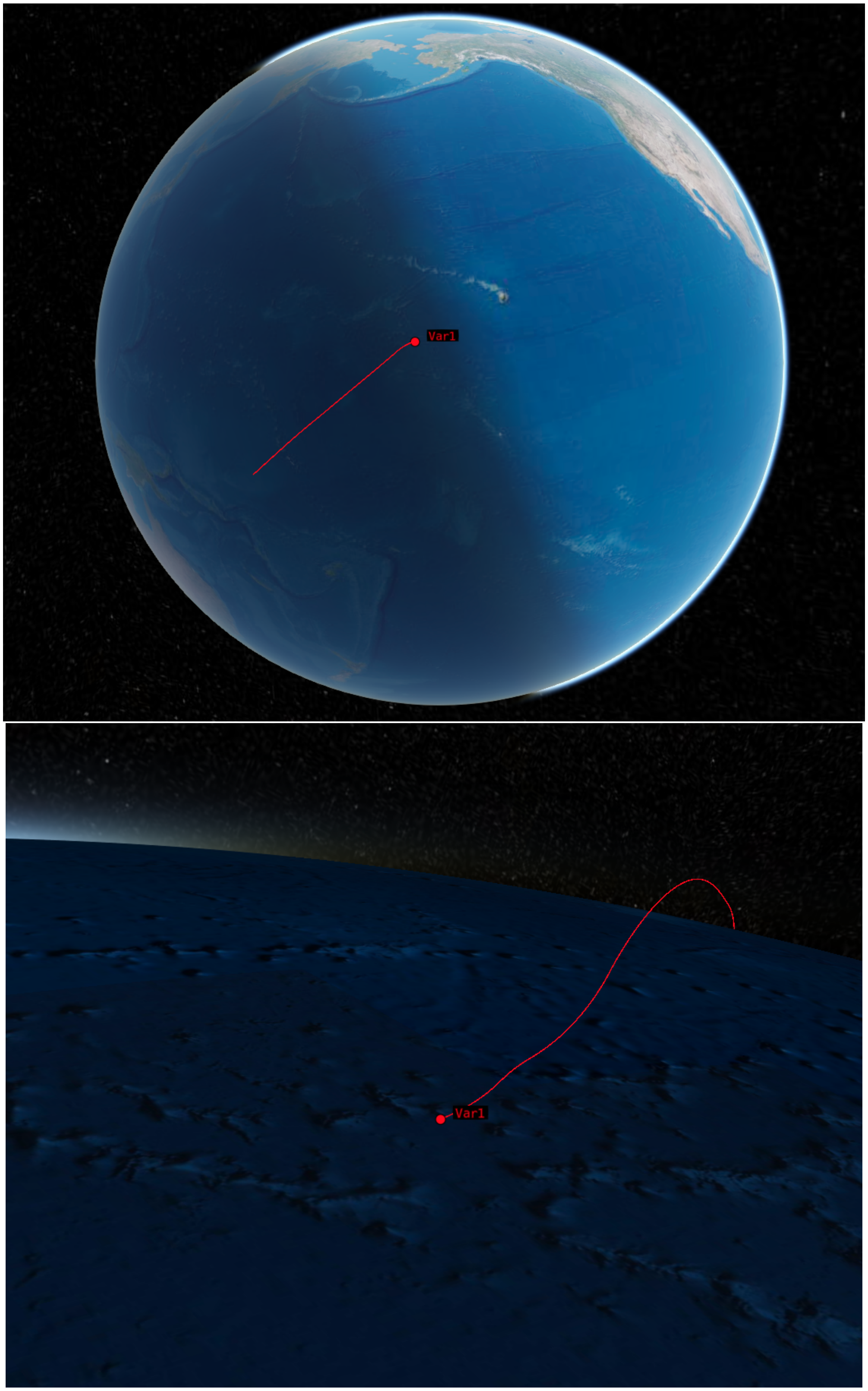

Fig. 6 Trajectory from simulation plotted in MATLAB visualization tool.

IV. Verification

From reference [2], we can verify our simulation data by providing a plot of altitude versus ground elapsed time. This flight data plot shows a very similar altitude trajectory to the simulation data from the previous section. From Fig. 7, the altitude at the peak of the UPCONTROL phase is approximately $225,000 ft$. From Fig. 3, the same peak of the UPCONTROL phase occurs at 70.93 km or $232,000 ft$.

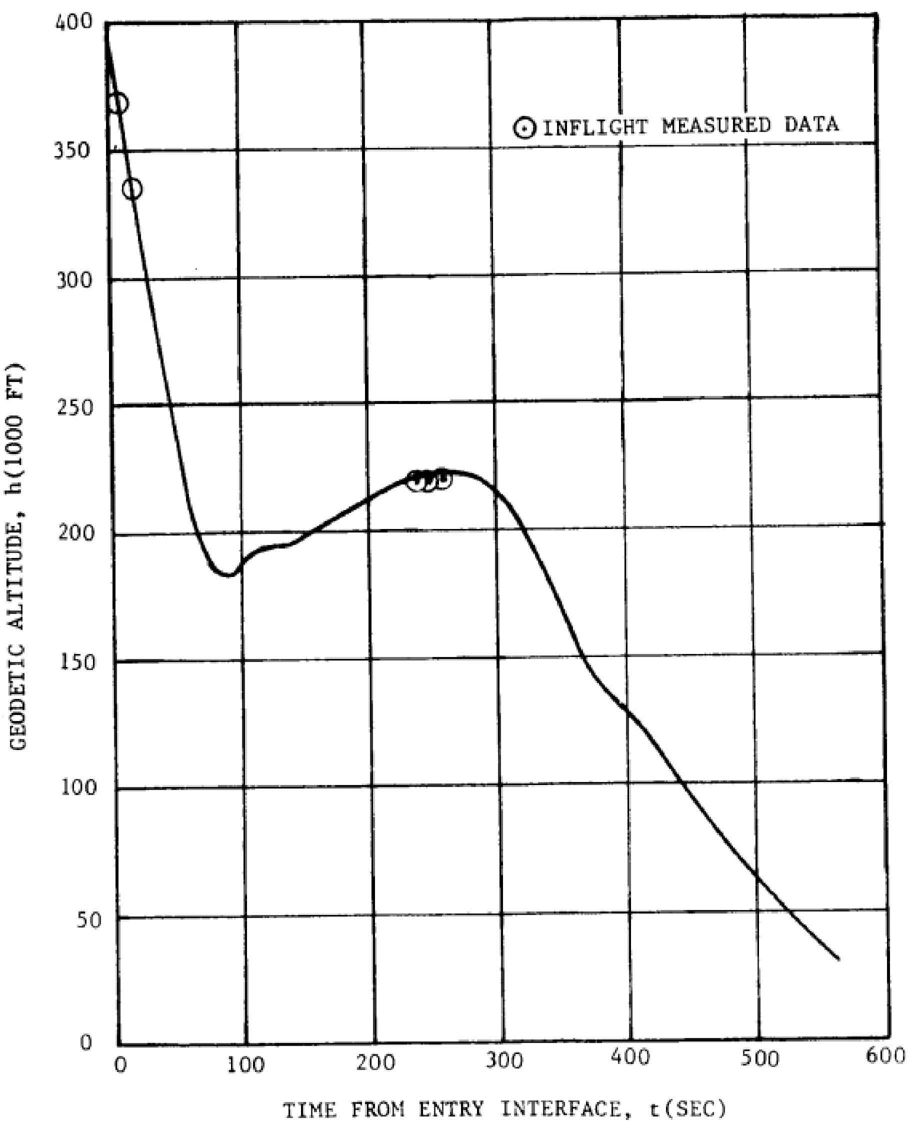

Fig. 7 Apollo 11 altitude flight data from [2].

The next few plots are from Apollo 4 [3] because of a lack of data obtainable from Apollo 11. In Fig. 8 is a plot of the flight path angle, Earth relative velocity, and heading angle of the Apollo 4 CM. The flight path angle plot is very similar to the simulation plot from Fig. 5. Also, the Mach number plot from Apollo 4 in Fig. 9 is very similar to the Mach number plot from the simulation in Fig. 3. The coordinates for the landing site of the Apollo 11 CM are 169.15 (deg West) and 13.30 (deg North). In our simulation we did not consider the parachute phase, so the CM was still approximately 10 km in altitude. However, at the end of our simulation the coordinates were 168.16 (deg West) and 14.38 (deg North). This is an approximate error of 1 degree in longitude and latitude. This is an approximate error of 75 nautical miles. In our simulation we did not consider the target coordinates and only provided the bank angle profile which is why we see a large deviation in the target coordinates. If a more sophisticated control technique was used, then we would have a much closer landing to the target coordinates.

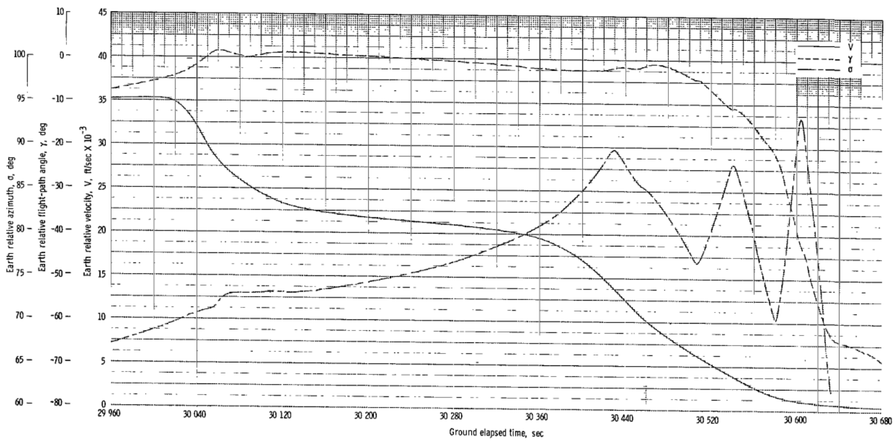

Fig. 8 Apollo 4 flight data from [3].

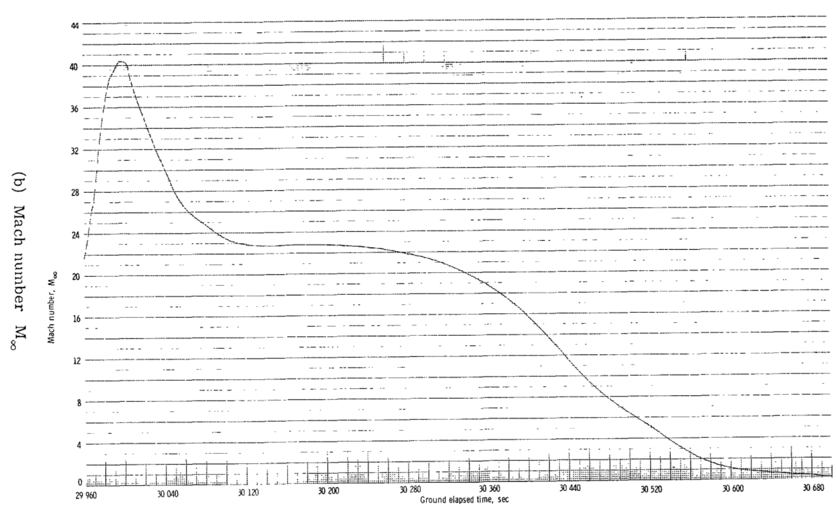

Fig. 9 Apollo 4 flight data from [3].

V. Conclusion

This report provides a simulation of the Apollo 11 command module Earth entry up to the end of the guidance phase. We demonstrated that with a closed-loop control on the commanded bank angle from the Apollo 11 flight data we were able to achieve close results to the original Apollo 11 flight. The simulated landing zone was approximately 75 nautical miles from the original Apollo 11 landing coordinates. With an onboard simulation of the predicted landing zone and correction, we can achieve smaller error in the splashdown coordinates.

References

[1] William C. Moseley., R. H. M., and Hughes, J. E., "STABILITY CHARACTERISTICS OF THE APOLLO COMMAND MODULE," Tech. rep., National Aeronautics and Space Administration, Mar. 1967.

[2] Manders, R., "APOLLO 11 POSTFLIGHT ANALYSIS," Tech. rep., National Aeronautics and Space Administration, Feb. 1970.

[3] Hillje, E. R., "ENTRY AERODYNAMICS AT LUNAR RETURN CONDITIONS OBTAINED FROM THE FLIGHT OF APOLLO 4," Tech. rep., National Aeronautics and Space Administration, Oct. 1969.